Comment on "Robustness of a Local Fermi Liquid against Ferromagnetism and Phase Separation"

In a Letter [1], on which ongoing research is based [2], Engelbrecht and Bedell (EB) studied the properties of Fermi liquids with a local (i.e., $\mathbf{k}$-independent) irreducible self-energy. Two of their main results were that such *local Fermi liquids* are robust against ferromagnetism and phase separation. In this Comment we want to point out that the conclusions of [1] are *not* generally valid for Fermi liquids with a local self-energy. We show that the conclusions of EB are *not* valid for lattice models in high spatial dimensions ($d \to \infty$), which are the only systems known to date for which the self-energy is purely local. We argue that the authors' statement, that "the dynamical mean-field theories, which become exact in infinite d, should lead to results that are compatible with ours $(\cdots)$", does not hold.

We start by recalling some of the properties of lattice Fermi systems in $d = \infty$. In high spatial dimensions the self-energy $\Sigma(\mathbf{k})$ is *local* [3]. Thus it can be seen as a functional of the local Green function: $\Sigma = \Sigma[G_{\rm loc}(\omega)]$, where $G_{\rm loc}(\omega) = \mathcal{N}^{-1}\sum_{\mathbf{k}} G(k)$ and $\mathcal{N}$ is the number of lattice sites. *However*, the *irreducible* [4] vertex functions are *not local*. Their momentum dependence enters explicitly through the parameter $x(\mathbf{k}) = d^{-1}\sum_{i=1}^{d}\cos(k_i)$ [5], e.g., the three contributions to the irreducible vertex function in second order in a local interaction are for $d \to \infty$ dependent on $x(\mathbf{p}-\mathbf{p}')$ (bubble), $x(\mathbf{p}-\mathbf{p}')$ (parallel interaction lines) or $x(\mathbf{p}+\mathbf{p}'+\mathbf{q})$ (crossing interaction lines).

EB assume that a local self-energy $\Sigma = \Sigma[G_{\rm loc}(\omega)]$ *implies* a local irreducible vertex function: $\Gamma^{\rm IR} = \delta\Sigma(\omega)/\delta G_{\rm loc}(\omega')$ (below eq. (4) in [1]). But even in the limit $d \to \infty$, where the assumption of a local self-energy is best, the locality of the self-energy does *not* imply the locality of the irreducible vertex function, see above. This means that functional derivation and limit process do not commute. By extension, an almost local self-energy in finite dimensions does *not* imply an almost local irreducible vertex function. Based on this crucial point, we argue that EB's concept of a strictly local Fermi liquid is not realistic for finite dimensions (e.g. $d = 3$).

Our second point (see also [6]) is that EB assume *isotropy* of the Fermi surface. This assumption is manifest in their eqs. (3) and (5). However, for 2- and 3-dimensional lattice models, like the Hubbard model, the Fermi surface deviates strongly from a sphere near half-filling. In $d = \infty$ the anisotropy is particularly drastic. One finds, e.g., that major parts of the Fermi "sphere" are chopped off for $d \to \infty$ [3] and that the average angle between the radial direction $\mathbf{p}$ and the normal direction $\mathbf{v} = \boldsymbol{\nabla}\varepsilon(\mathbf{p})$ is approximately 38.8° [7]. We conclude that eq. (6) in [1] is inappropriate for lattice models.

Thirdly, the argument in [1] suggesting robustness of the local Fermi liquid against phase separation is not sufficient. EB apply the "Pomeranchek criterion" (that the compressibility be positive) only to the Fermi liquid state, disregarding phases with broken symmetries. However, phase separation is a *first order* transition which cannot be determined from the local behavior of the compressibility, as suggested in [1]. Instead, the Pomeranchek criterion should be applied globally (to all possible phases), before phase separation can be excluded.

Furthermore, the robustness predicted by EB against ferromagnetism disagrees with the manifestation of this phase in work on $d = \infty$ Hubbard models on certain lattices [6,8,9]. Two examples in $d = \infty$, displaying a first order instability towards phase separation, are the model for interacting spinless fermions [10] and the Hubbard model [11].

We conclude that the assumptions in [1], isotropy of the Fermi surface and locality of the vertex function, are not realized in $d = \infty$ and cannot be considered realistic in $d = 3$. The result in [1], that local Fermi liquids are "robust" against ferromagnetism and phase separation, is not generally valid.

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